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ETAT DE L'ART SUR LES PROBLEMES D'AFFECTATION DES DELAIS ET D'ORDONNANCEMENT : LE CAS DU DELAI COMMUN

Valery GORDON*, Jean-Marie PROTH** and Chengbin CHU***

Résumé :

L'objectif de cette communication est de fournir une approche unifiée pour les problèmes d'affectation de délais communs et pour les ordonnancements correspondants en partant de la littérature relative aux systèmes à machine unique et à machines parallèles. Nous nous attachons à la situation dite "statique" dans laquelle l'ensemble des tâches est connu au moment où débute les calculs, par opposition aux situations dites "dynamiques" dans lesquelles les tâches apparaissent tout au long du temps et doivent être traitées en temps réel.

Les problèmes auxquels nous nous intéressons ont pris une grande importance au cours des dix dernières années du fait de l'apparition de nouvelles méthodes de gestion des stocks, comme par exemple la méthode du "Juste-à-Temps".

Le modèle à délai commun, qui est également connu dans la littérature sous le nom de "CON model", où CON remplace "CONstant flow allowance" s'applique, par exemple, à un système d'assemblage dans lequel les composantes doivent être prêtes en même temps, ou à un magasin d'expédition où plusieurs produits, qui font partie de la même commande, doivent être disponibles en même temps.

Dans les problèmes que nous examinons, l'objectif est de déterminer un délai et l'ordonnancement qui permet de l'atteindre afin d'optimiser un critère fonction du délai et des temps de fin d'exécution des tâches.

Les algorithme et leur complexité sont résumés dans un tableau.

MOTS CL2S : Affectation des délais, Ordonnancement, Etat de l'art, Manufacturing, Complexité.

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A State-of-the-Art Survey of Due Date Assignment and Scheduling Research: Common Due Date

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ABSTRACT

In this paper, we aim at providing a unified framework of the common due date assignment and scheduling problems in the deterministic case by surveying the literature concerning the models involving single machine and parallel machines. We focus on static production settings in which a fixed set of jobs is available for processing as opposed to dynamic production settings where jobs continuously arrive in the system and should be scheduled on-line. The problems with due date determination have received considerable attention in the last ten years due to the introduction of new methods of inventory management such as Just-In-Time (JIT) systems. The common due date model which is also known in scheduling literature as CON model, where CON stands for *constant* flow allowance, corresponds, for instance, to an assembly system in which the components of the product should be ready at the same time, or to a shop where several jobs constitute a single customer's order. In the problems under consideration, the objective is to find an optimal value of the common due date and the related optimal schedule in order to optimize a given criterion based on the due date and the completion times of jobs. The results on the algorithms and complexity of the common due date assignment and scheduling problems are summarized.

KEYWORDS

Due date assignment, Scheduling, State-of-the-art, Manufacturing, Complexity.

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A STATE-OF-THE-ART SURVEY OF DUE DATE ASSIGNMENT AND SCHEDULING RESEARCH: COMMON DUE DATE

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The scheduling problems involving due dates are of permanent interest. One of the first scheduling investigations undertaken by Jackson (1955) was the starting point of this type of research. The subsequent publications of survey papers on specific aspects of due date scheduling problems as, for instance, the works of Sen and Gupta (1984), Raghavachari (1988), Cheng and Gupta (1989), Baker and Scudder (1990), Koulamas (1994), confirm this interest. The problems with the objective functions involving lateness, tardiness and earliness with respect to the given due dates took up a significant place in the recent reviews on scheduling problems presented by Gupta and Kyparisis (1987), Kawaguchi and Kyan (1988), Cheng and Sin (1990), Lawler *et al.* (1993), Hoogeveen, Lenstra and van de Velde (1997). The scheduling books which have been published in the 1990's also contain extensive lists of references for the scheduling and sequencing problems with due dates as, for instance, Blazewicz *et al.* (1993), Morton and Pentico (1993), Tanaev, Gordon and Shafransky (1994), Tanaev, Sotskov and Strusevich (1994), Brucker (1995), Chretienne *et al.* (1995), Pinedo (1995). But there is only one survey by Cheng and Gupta (1989) where the due date assignment decisions are of primary interest. These problems are also partially considered by Baker and Scudder (1990) who gave a comprehensive review of scheduling problems with non-regular performance measure, but both reviews cover only the results published before the 1990's.

In this paper, we aim at providing a unified framework of the due date assignment problems in the deterministic case by surveying the literature concerning the models involving single machine and parallel machines. We pay a particular attention to the single and parallel machine shops since there are quite few results on open, flow and job shop due date assignment.

We focus on static production settings in which a fixed set of jobs is available for processing as opposed to dynamic production settings where jobs continuously arrive in the system and should be scheduled on-line. In particular, we consider the models which concern the scheduling of n jobs (the activities) on m machines (the resources), where a machine can process at most one activity at a time, and the objective is to optimize some function of the job completion times and the due dates.

In early researches, the computer simulation techniques were applied to determine the better due dates (Conway, 1965; Eilon and Chowdhury, 1976; Weeks and Fryer, 1979). An overview of the studies which examine (using simulation) the relative performance of simple heuristic rules for the due date assignment can be found in the survey by Cheng and Gupta (1989).

Seidmann, Panwalkar and Smith (1981) and Panwalkar, Smith and Seidmann (1982) were the first who considered the optimal due date assignment problems together with scheduling decisions and tackled the problems analytically. They noted that job scheduling can be approached either from the viewpoint of the shop manager or from the viewpoint of the customer placing the job orders. When the shop viewpoint is taken, the objective of the scheduling is usually related to the minimization of one or more cost functions associated with the state of the shop (work-in-process, machine load, etc.), while it is more likely related to the due date from the customer viewpoint.

The due date assignment problems make practical sense when a firm offers a due date to its customers during sale negotiations and has to offer a price reduction when the due date is far away from the expected one. In many instances, due dates are negotiated rather than simply dictated by the customers. The later the due dates are fixed, the higher the probability that the product will be completed or delivered on time. In order to maintain a good image among the customers, many companies tolerate reasonable holding costs in favour of keeping the established due dates. Thus, the decision maker has to balance the losses resulting from the holding costs and the advantages of fulfilling the orders in time.

The problems with due date determination have received considerable attention in the last ten years due to the introduction of new methods of inventory management such as Just-In-Time (JIT) systems. T.C.E. Cheng, who contributed a lot to the due date assignment and the related scheduling approaches, remarks that completing a job early means to bear the costs of holding unnecessary inventories, while finishing a job late results in contractual penalty and loss of customer goodwill.

Let d_j be the due date of job j (to be assigned). Several models of assigning due dates have to be taken into consideration. The simplest is the model in which all jobs have a common due date (i. e., $d_j = d$, $j = 1,...,n$). Such a model corresponds, for instance, to an assembly system in which the components of the product should be ready at the same time, or to a shop where several jobs constitute a single customer's order. More generally, this model corresponds to any system in which, for some reason (appointment, technical constraints, etc.), several tasks should be completed at the same time. This method of due date assignment is known in scheduling literature as CON model, where CON stands for *constant* flow allowance. It is also called the common due date model.

If each job j has a release date r_j (earlier time for processing), the CON due date is defined as $d_j = r_j + d$. Here, d may be considered as the common production cycle for all the products which should be optimized.

In the SLK due date assignment (SLK stands for *slack*), a flow allowance that reflects equal waiting time or slack, denoted by q, is assigned to the jobs. Thus, $d_j = r_j + p_j + q$ or, when all jobs have the same release date, $d_j = p_j + q$. In this model, the goal is to find an optimal value of the slack q with respect to the criterion to be optimized.

In the TWK due date assignment (TWK stands for *total work*), due dates are based on total work content and are equal to a multiple of the job processing times, i. e., $d_j = kp_j$, or, when the release dates are given, $d_j = r_j + kp_j$. The optimal due date assignment consists of finding the value of the common multiplier k.

The PPW due date assignment (*Processing Plus Wait*) combines SLK and TWK due date assignment rules. In this model, $d_j = kp_j + q$ for simultaneously available jobs, or $d_j = r_j + kp_j + q$ when the release dates are different. The optimal due date assignment consists of finding the optimal values of k and q.

In the NOP (*number of operations*) model, due dates are determined on the basis of the number of operations n_j to be performed on job *j*: $d_j = r_j + kn_j$. The goal, in this case, is to optimize *k*.

In a recently identified class of generalized due date scheduling problems (Hall, 1986a; Hall, Sethi and Sriskandarajah, 1991), the set of due dates is given, and the problem consists in assigning the jobs to the due dates. When k jobs should be performed by the *k*th due date, such due dates may be called positional due dates (Hoogeveen, Lenstra and van de Velde, 1997). For instance, if three due dates d_1, d_2, d_3, where $d_1 < d_2 < d_3$, are given for the set of three jobs, the objective would be to complete at least one job by d_1, at least two jobs by d_2, and all three jobs by d_3. This method of due date assignment describes the situation where what matters is *how many* jobs have been completed by any point in time, rather than *which* jobs they are.

In this paper, we do not consider the RDM (*random* allowance) method, where each job receives a random due date, usually following a probability distribution, or the JIQ (*jobs in queue*) and the JIS (*jobs in system*) methods, where due dates are determined based on, respectively, the current job queue length and the number of jobs in the system. These due date assignment rules are usually studied by simulation models. Some recent results related to these approaches may be found in Ramasesh (1990), Vig and Dooley (1991, 1993), Gee and Smith (1993), Smith, Minor and Wen (1995), Lawrence (1995), Roman and Valle (1996), Philipoom, Wiegmann and Rees (1997), Tsai, Chang and Li (1997).

The following notations are used in the rest of the paper.

If C_j, E_j and T_j denote respectively the completion time, the earliness and the tardiness of job *j*, then

$$E_j = max\{0, d_j - C_j\} \text{ and } T_j = max\{0, C_j - d_j\}. \tag{1}$$

Lateness L_j of job j is defined as $L_j = C_j - d_j$. Hence, $E_j = max\{0, -L_j\}$ and $T_j = max\{0, L_j\}$.

This paper is organized as follows. In Sections 1 and 2, respectively, we consider the CON (common due date) model for the single machine and for parallel machines. In this case, all the jobs have a common due date d which is to be assigned. The objective is to find an optimal value of this due date and the related optimal schedule in order to optimize a given criterion based on the due date and the completion times of jobs. In Section 3, the results on complexity of the common due date assignment and scheduling problems are summarized. The

SLK, TWK, NOP and PPW due date assignment models as well as the positional due dates will be considered in another paper which may be regarded as the second part of this paper.

1. SINGLE MACHINE

Consider the problem of scheduling n jobs on a single machine. A processing time p_j is associated with each job $j, j=1,...,n$, all jobs are available at the same time (i.e., the release date is the same for each job), and preemption is not allowed. Let d be a due date, and $\sigma = ([1],[2],...,[n])$ be an arbitrary sequence of the jobs where $[j]$ is the jth job in σ. Then, $E_{[j]} = max\{0, d - C_{[j]}\}$ is the earliness of the jth job for the due date d in σ, and $T_{[j]} = max\{0, C_{[j]} - d\}$ is the tardiness of this job.

Let us introduce some additional notations useful for further explanations. Assuming (throughout this section) that the jobs are labelled in the increasing order of their processing times, we define $B = \{n, n-2,...,1\}$, $A = \{2,4,...,n-1\}$ when n is odd, and $B = \{n, n-2,...,2\}$, $A = \{1,3,...,n-1\}$ when n is even. Let $\Delta = \sum_{j \in B} p_j$.

1.1. In the case where the due date is given, the problem usually considered in the literature aims at minimizing the mean absolute deviation of completion times about a common due date (MAD) or, which is the same, at minimizing the mean absolute lateness. The MAD problem is defined as follows: find a sequence σ^* and a starting time t^* to minimize the penalty function given by $MAD(\sigma) = 1/n \sum_{j=1}^{n} \left| C_{[j]} - d \right|$. Since the constant $1/n$ may be removed without loss of generality, we will consider later on $MAD(\sigma) = \sum_{j=1}^{n} \left| C_{[j]} - d \right|$. This objective function may be easily rewritten in the following way: $MAD(\sigma) = \sum_{j=1}^{n} (E_{[j]} + T_{[j]})$.

Kanet (1981) showed that, for a given common due date $d \geq \sum_{j=1}^{n} p_j$, the schedule (B, A) with the starting time $t = d - \Delta$ is optimal for the MAD problem.

Bagchi, Sullivan and Chang (1986) showed that Kanet's result holds for any due date $d \geq \Delta$ and gave a procedure for finding all optimal solutions of the problem. Raghavachari (1986) showed that for any value of the due date d, an optimal sequence for this problem is V-shaped. A V-shaped sequence is such that a subset of jobs placed in the non-increasing order of the p_j values (longest processing time order, known as LPT) is followed by the remaining jobs in the non-decreasing order of the p_j values (shortest processing time or SPT order).

1.2. Panwalkar, Smith and Seidmann (1982) were the first to consider the MAD problem as a due date assignment problem. Actually, they consider the problem with a more general objective function than MAD, namely, the problem of finding a sequence σ^* and a common due date d^* which lead to the minimal value of

$$f(d,\sigma) = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + \gamma d),$$

where α, β and γ are non-negative constants. When $\gamma = 0$ and $\alpha = \beta = 1$ we have the MAD problem where due date is to be assigned.

It is easy to show (Panwalkar, Smith and Seidmann, 1982) that in the case where $\gamma \geq \beta$, the SPT order is optimal and $d^* = 0$. If $\gamma < \beta$, the following properties lead to a polynomial-time algorithm (Panwalkar, Smith and Seidmann, 1982) for this problem.

Property 1. For any specified sequence σ, there exists an optimal value of d which coincides with a completion time of one of the jobs in σ.

Property 2. For any specified sequence σ, there exists an optimal due date equal to $C_{[K]}$, where K is the smallest integer value greater than or equal to $n(\beta - \gamma)/(\alpha + \beta)$, and exactly K jobs will be non-tardy.

So, the total penalty is equal to $f(C_{[K]},\sigma)$ and, introducing $C_{[K]} = p_{[1]} + p_{[2]} + \ldots + p_{[K]}$, we get

$$f(C_{[K]},\sigma) = \sum_{j=1}^{K}(n\gamma + (j-1)\alpha)p_{[j]} + \sum_{j=K+1}^{n}\beta(n+1-j)p_{[j]} = \sum_{j=1}^{n}a_j p_{[j]},$$

where a_j is equal to $n\gamma + (j-1)\alpha$ for $j \leq K$ and to $\beta(n+1-j)$ for $j > K$. To find the optimal sequence, we need to find the minimal penalty among all sequences σ. The term a_j may be viewed as the "positional" penalty for the job in position j in σ or, in other words, the penalty resulting from the job in position j.

Property 3. Quantity $\sum_{j=1}^{n}a_j p_{[j]}$ is minimized by matching the smallest value of a with the largest value of p, the next larger value of a with the next smaller value of p, and so on.

This property leads to an $O(n\log n)$ algorithm. It finds first the number of non-tardy jobs (K), and then calculates the positional penalties a, the optimal sequence σ^* and the optimal due date d^*. The sequence generated by this algorithm is V-shaped. The optimal schedule starts at time $t^* = 0$ and has no inserted idle periods.

Cheng (1989) noted that if $K = n(\beta-\gamma)/(\alpha+\beta)$, it is not necessary to constrain d^* to be equal to one of the job completion times. The optimal due date can be set at $d^* = C_{[K]} + \delta p_{[K+1]}$, $0 \le \delta < 1$. Thus, in this case, there exists an infinite number of optimal due dates. For example, if $\gamma = 0$, $\alpha = \beta$ and n is even, then $K = n/2$ and any value between $C_{[n/2]}$ and $C_{[(n/2)+1]}$ is an optimal due date.

1.3. Returning to the MAD problem with due date assignment (the special case of the considered problem with $\gamma = 0$, $\alpha = \beta = 1$), we derive from Property 2 that the optimal due date is equal to the completion time of the Kth job, where K is the smallest integer value greater than or equal to $n/2$, i. e., $K = n/2$ if n is even, and $K = (n+1)/2$ if n is odd.

The positional penalty a for the job in position j for the MAD problem is $j-1$ if $j \le K$ and $n+1-j$ if $j > K$. Hence, the penalties are the same for positions 2 and n, for 3 and $n-1$, for 4 and $n-2$, and so on. Therefore, according to the Property 3, there are two ways of matching values of a with processing times and hence of assigning jobs to the positions of the optimal sequence. As a consequence, the total number of optimal sequences (when the processing times are unique) is 2^r, where $r = (n-1)/2$ if n is odd, and $r = n/2$ if n is even (Bagchi, Sullivan and Chang, 1986; Hall, 1986b; Karacapilidis and Pappis, 1995a). Each of these schedules (sequences) has its own optimal due date, equal to $C_{[K]}$.

So, there may be different optimal due dates and different optimal sequences. Moreover, increasing d^* by a given value $t > 0$ and increasing the starting times of the jobs in the optimal sequence by the same value t leads to another optimal solution (Alidaee, Kochenberger and Ahmadian, 1994). Dickman, Wilamowsky and Epstein (1991) noted that, in the case when n is even, all due dates from the interval $[C_{[n/2]}, C_{[(n+1)/2]}]$ are optimal.

Bagchi, Chang and Sullivan (1987) found that the minimal completion time of the Kth job in the optimal schedule (i. e., the minimum sum of processing times of non-tardy jobs) is Δ. So, the minimal value of optimal due date is $d^* = \Delta$.

The complexity of the MAD problem changes drastically when the admissible due dates are upper bounded by D with $D < \Delta$. Such a problem is called restricted common due date problem as opposed to the unrestricted problems where $d \ge \Delta$ (note that, in the unrestricted problem, the optimal cost cannot decrease when the common due date increases). For the restricted problem, Properties 1-3 above do not hold anymore, and the optimal solution may contain a straddling job which starts before d and is completed after d (but there still exists an optimal schedule which is V-shaped). The following useful property holds for this problem.

Property 4. Either there exists a job in the optimal schedule that completes at the due date, or the schedule starts at time zero.

Hall, Kubiak and Sethi (1991) and Hoogeveen and Van de Velde (1991) proved that the restricted MAD problem is NP-hard.

For this problem, Bagchi, Sullivan and Chang (1986) proposed an algorithm (enumerative in nature, which is effective at solving problems up to 15 jobs in size) under the implicit assumption that the starting time of the schedule is zero. Note that the introduction of this assumption contradicts the common sense: due to Property 4, the best solution may include the delayed start (Szwarc, 1989). A sufficient condition (depending on the processing times and due date) for a schedule to be optimal when starting at time zero was given by Szwarc (1989), as well as a branch-and-bound procedure to deal with the problems up to 25 jobs in size. In the case of a zero starting time, Sundararaghavan and Ahmed (1984) proposed a heuristic procedure. Hall, Kubiak and Sethi (1991) gave an $O(n\sum p_j)$ algorithm without the zero starting time assumption. This pseudopolynomial-time algorithm allows to deal with the problems up to 1000 jobs. Ventura and Weng (1995) showed that two out of the three subroutines in the algorithm proposed by Hall, Kubiak and Sethi (1991) can be eliminated and, consequently, that the total computational effort can be reduced. Hoogeveen, Oosterhout and van de Velde (1994) presented a branch-and-bound algorithm based on Lagrangean lower and upper bounds (which is effective at solving problems up to 50 jobs in size) and an $O(n\log n)$ 4/3-approximation algorithm for the MAD problem. Performance guarantee 4/3 means that, for any instance, the approximation algorithm produces a solution with objective function value at most 4/3 times the optimal value.

Hall, Kubiak and Sethi (1991) also gave an $O(n\log n)$ algorithm for the special case of the restricted MAD problem, where there exists $l \geq \lceil n/2 \rceil$ satisfying $\sum_{j=1}^{l} p_j < d$ and $p_{l+1},\dots,p_n > 2d$. Another special case solvable in $O(n\log n)$ time is obtained when d is so small that, whichever the schedule considered, no job is early (Bagchi, Sullivan and Chang, 1986).

1.4. Let us now consider the problem (derived from the one studied by Panwalkar, Smith and Seidmann (1982) when $\gamma = O$), where α and β may be different. The goal is to minimize the weighted sum of absolute deviation (WSAD) of the completion times with regard to a common due date. The objective function for the WSAD problem may be written as:

$$WSAD(d,\sigma) = \alpha \sum_{j\in E}|C_j - d| + \beta \sum_{j\in T}|C_j - d| = \alpha \sum_{j\in E}(d - C_j) + \beta \sum_{j\in T}(C_j - d) =$$
$$\sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]}),$$

where $E = \{j|C_j \leq d\}$ is a set of jobs that are not tardy and $T = \{j|C_j > d\}$ is a set of jobs that are tardy.

Like the MAD problem in **1.3**, this problem has different optimal due dates as well as an infinite number of optimal solutions obtained by increasing the optimal due date and the starting times of the jobs by the same value. Bagchi, Chang and Sullivan (1987) gave an $O(n\log n)$ algorithm alternative to the matching procedure of Panwalkar, Smith and Seidmann (1982). This algorithm yields a unique sequence with the minimal value of the due date among the optimal due dates.

Panwalkar and Rajagopalan (1992) considered the WSAD problem with controllable processing times. The processing time p_j of job j may be chosen in the interval $[t'_j, t_j]$, where $p_j = t_j$ is the normal time but p_j can be reduced as long as it remains greater than t'_j. In this case, the cost incurred by such reduction is λ_j per time unit. If x_j is the actual amount of time by which job j is compressed, then $\lambda_j x_j$ represents the processing cost of job j. The objective is to find a set of processing times, a sequence and the smallest due date to minimize the cost function

$$f(d,\sigma) = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + \lambda_{[j]}x_{[j]}).$$

Panwalkar and Rajagopalan (1992) showed that there exists an optimal sequence such that no jobs will be partially compressed ($x_j = 0$ or $x_j = t_j - t'_j$), and proposed an $O(n^3)$ algorithm to find the optimal solution by formulating the problem as an assignment problem.

The restricted WSAD problem (when the values of due date have a given upper bound D and $D < \Delta$) is NP-hard due to NP-hardness of the restricted MAD problem. For this problem, Bagchi, Chang and Sullivan (1987) proposed branching procedure suitable for the problems up to 15 jobs in size.

1.5. A natural generalization of MAD and WSAD problems is the problem which consists in minimizing the total weighted earliness and tardiness (TWET) about a common due date. This problem may be formulated as a due date assignment and scheduling problem in the following way: find a sequence σ^* and a common due date d^* to minimize the objective function

$$TWET(d,\sigma) = \sum_{j\in E}\alpha_j(d-C_j) + \sum_{j\in T}\beta_j(C_j-d) = \sum_{j=1}^{n}(\alpha_j E_j + \beta_j T_j),$$

where for job j, $\alpha_j > 0$ is a unit earliness penalty (weight) and $\beta_j > 0$ is a unit tardiness penalty (weight), E and T are defined as in **1.4**. If $\alpha_j = \beta_j$ for each $j=1,\ldots,n$, the earliness and tardiness penalty weights are said to be symmetric.

Hall and Posner (1991) proved that the TWET problem is NP-hard even in unrestricted setting with symmetric weights. They proposed an $O(n\sum p_j)$ dynamic programming algorithm that solves to optimality large instances of the problem. Hoogeveen and Van de Velde (1991) gave an $O(n^2\sum p_j)$ algorithm for the restricted variant of this problem. Since these algorithms are pseudopolynomial, the TWET problem with symmetric weights is NP-hard in the ordinary sense.

Cheng (1985, 1987a), Quaddus (1987), Bector, Gupta and Gupta (1988), Backer and Scudder (1989), Hall and Posner (1991) investigated the properties of the unrestricted TWET problem. Most of these properties are similar to Properties 1 - 2 in **1.2** and can be formulated in the following way.

Property 5. There exists an optimal schedule which is:

a) V-shaped in the sense that the jobs in set E (non-tardy jobs) are sequenced in the non-increasing order of the ratio p_j/α_j and followed by the jobs of set T (tardy jobs) in the non-decreasing order of the ratio p_j/β_j,

b) the optimal due date coincides with the completion time of the last non-tardy job in an optimal sequence, and

c) $\sum_{j\in T}\beta_j \le \sum_{j\in E}\alpha_j$.

Property 5a is valid also for the restricted problem, however 5b and 5c do not hold in the restrictive case.

Hall and Posner (1991) developed a fully polynomial approximation scheme for the unrestricted TWET problem with symmetric weights when the maximum weight is bounded by a polynomial function of n. Such schemes produce a heuristic solution close to the optimal one and represent a family of algorithms $\{A_\varepsilon\}$ with the following properties. For each $\varepsilon > 0$, A_ε is a polynomial-time $(1+\varepsilon)$-approximation algorithm with the running time bounded by a polynomial not only in input size but also in $1/\varepsilon$. Remind that, for a minimization problem f, a

ρ-approximation algorithm ($\rho > 1$) delivers a solution with a value of the objective function at most $\rho f(x)$ for each input x.

Cheng (1990) presented an $O(n^{1/2}2^n)$ partial search algorithm for determining an optimal sequence and the corresponding common due date for the unrestricted TWET problem with symmetric weights. Quaddus (1987) considered the separate earliness and tardiness penalties for each job ($\alpha_j \neq \beta_j$) and showed that, once the job sequence is given, an optimal due date can easily be assigned. To find both the optimal due date and the optimal sequence in the case when $\alpha_j \neq \beta_j$, branching procedures (Dileepan, 1993; De, Ghosh and Wells, 1994a) and heuristic algorithms (Gupta, Bector and Gupta, 1990; Dileepan, 1993; De, Ghosh and Wells, 1994a) were proposed. The most convincing results of computational experiment are due to De, Ghosh and Wells (1994a), who proposed a greedy randomized adaptive search heuristic procedure. The procedure consists in two phases: an initial solution is constructed in the first phase through controlled randomization, and is improved in the second phase through steepest descent neighbourhood search.

James (1997) proposed to use tabu search for both restricted and unrestricted TWET problems. Hao *et al.* (1996) considered the common due date determination and sequencing using tabu search for the TWET problem with symmetric weights.

Lee, Danusaputro and Lin (1991) presented a pseudopolynomial time procedure for the special case of the TWET problem with asymmetric weights when the jobs have agreeable ratios, which means that $p_i/\alpha_i < p_j/\alpha_j$ implies $p_i/\beta_i < p_j/\beta_j$ for all i and j. Szwarc (1996) proposed several improvements of this procedure.

Hall and Posner (1991) founded some polynomially solvable special cases of the unrestricted TWET problem. In particular, an $O(n)$ algorithm solves the problem with $\alpha_j = \beta_j = p_j$, $j = 1,...,n$, and an $O(n\log n)$ algorithm solves the problem with $\alpha_j = \beta_j, p_j = 1$; $j = 1,...,n$. Pappis and Adamopoulos (1993) proposed an algorithm in case of $\alpha_j = \lambda p_j^a$, $\beta_j = \lambda p_j^b$, where $\lambda > 0$, a and b are non-negative integers. For the problem where weights are proportional to the processing times of the corresponding jobs ($\alpha_j = \beta_j = \lambda p_j$, $\lambda > 0$, $j = 1,...,n$), Karacapilidis and Pappis (1995b) proposed an algorithm to find $r!(n-r)!$ alternative optimal sequences, where r is a position of job whose completion time coincides with the optimal due date. The optimal due date and each sequence are found in polynomial time.

Hoogeveen and Van de Velde (1991) studied the following polynomially solvable special cases for the restricted TWET problem: a) $p_j = p$ for each job j, $j = 1,...,n$; b) $p_j = w_j$ for each job j, $j = 1,...,n$. For both cases, $O(n \log n)$ algorithms were proposed.

1.6. Cheng (1988) was the first who considered common due date assignment and scheduling problem by introducing a completion time deviation allowance e. This problem models the real life situation when penalty does not occur if the deviation of job completion time from the due date is small enough. Assuming that parameter e satisfies the condition $2e < min\{p_j | 1 \leq j \leq n\}$, Cheng (1988) proposed an $O(n \log n)$ algorithm to find an optimal due date and an optimal sequence for the objective function

$$f(d,\sigma) = \sum_{j=1}^{n} (E_{[j]} U(E_{[j]} - e) + T_{[j]} U(T_{[j]} - e)),$$

where $U(x-e) = 0$ if $x \leq e$, and $U(x-e) = 1$ otherwise.

Weng and Ventura (1994) and Wilamowsky, Epstein and Dickman (1996) relaxed Cheng's restrictive requirement on parameter e and proposed polynomial-time algorithms in the case when no penalty costs are incurred for job completed within limits of any prespecified size around the common due date.

Unlike Cheng (1988) where, outside the tolerance interval, the penalty is proportional to the early or late amount with respect to d, Weng and Ventura (1994) considered a penalty reduced by $d-e$ compared to the one proposed by Cheng. As pointed out by Baker and Scudder (1990), such earliness-tardiness measurement is more conventional and consistent.

Dickman, Wilamowsky and Epstein (1991) discussed the availability of multiple optimal schedules.

Baker and Scudder (1990) generalized Cheng's approach to find the properties of an optimal schedule for the TWET due date assignment problem when completion time deviation allowances are different: job j avoids penalty if its completion time falls into the interval $[d-e_j, d+t_j]$ with the requirement that $p_j - t_j - e_j > 0$ for all pairs of jobs (i, j). The last inequality presupposed that the due date tolerance interval is relatively small compared to the processing times of the jobs.

1.7. Another way to take into account the situation, when only small deviations of the completion time from the due date are acceptable, is to consider the problem of minimizing the mean squared deviation (MSD) of job completion times about a common due date, i. e. the problem of minimizing the objective function

$$MSD(d,\sigma) = \sum_{j=1}^{n} (C_j - d)^2 = \sum_{j=1}^{n} (E_j^2 + T_j^2).$$

Being a quadratic penalty function, the MSD penalizes larger deviations at a higher rate.

For the unrestricted MSD problem (with $d \geq \sum_{j=1}^{n} p_j$), where an increase in the due date does not result in any further decrease in MSD, Bagchi, Sullivan and Chang (1987) found the following property.

Property 6. The optimal due date that minimizes the MSD for any given schedule is equal to the mean completion time $\overline{C} = (1/n)\sum_{j=1}^{n} C_j$.

Therefore, the unrestricted MSD problem is equivalent to the problem of minimizing completion time variance $CTV = \sum_{j=1}^{n} (C_j - \overline{C})^2$ which was considered by Merten and Muller (1972). The CTV problem was shown to be NP-hard by Kubiak (1993). Therefore, the unrestricted MSD problem is also NP-hard.

For the CTV problem, Gupta, Gupta and Kumar (1993) proposed a heuristic procedure based on genetic algorithms, and De, Ghosh and Wells (1992) presented a pseudopolynomial-time dynamic programming procedure.

It is worth to note that, for the unrestricted problem, MSD is invariant to any feasible linear translation of a schedule with its due date (the same as for the unrestricted MAD problem). Let us denote the optimal MSD by z^*. Then there exists a minimal due date d^* for which an MSD equal to z^* is the solution. Bagchi, Sullivan and Chang (1987) proposed a procedure based on the CTV problem to find d^*.

The restricted MSD problem is the problem where the values of due date have a given upper bound D with $D < \Delta$. Kahlbacher (1989, 1993) proposed an $O(n\sum p_j)$ algorithm for the restricted problem to minimize more general objective function, that is, $\sum_{j=1}^{n} (\alpha E_j^c + \beta T_j^c)$ for an arbitrary positive c. Enumerative procedures were proposed by Bagchi, Sullivan and Chang (1987) for the restricted MSD problem and by Bagchi, Chang and Sullivan (1987) for the restricted problem to minimize the weighted sum of squared deviations $\sum_{j=1}^{n} (\alpha E_j^2 + \beta T_j^2))$ with the assumption that schedule begins at time zero. This unnecessary assumption was removed by De, Ghosh and Wells (1989, 1990) in their enumerative procedure.

Using dynamic programming, De, Ghosh and Wells (1993) provided a comprehensive solution methodology with a pseudopolynomial complexity for the problems with objective functions $\sum_{j=1}^{n} (E_j^2 + T_j^2)$ and $\sum_{j=1}^{n} (\alpha E_j^2 + \beta T_j^2)$, as well as for the restricted MAD and WSAD problems considered in **1.3** and **1.4**.

1.8. Cai, Lum and Chan (1997) considered a general model, in which earliness and tardiness penalties are arbitrary non-decreasing functions. In this case, the problem consists in finding an optimal due date d^* and the corresponding sequence σ^* to minimize the objective function

$$f(d,\sigma) = \sum_{j\in E(\sigma)}(e_j + \alpha_j h(L_j)) + \sum_{j\in T(\sigma)}(t_j + \beta_j g(L_j)),$$

where e_j, t_j, α_j, β_j are non-negative, g and h are non-decreasing real-valued functions, $E(\sigma) = \{j|C_j < d\}$, $T(\sigma) = \{j|C_j > d\}$ and L_j is a lateness of job j. Here e_j (or t_j) represents a fixed cost incurred when job j is early (or tardy).

Cai, Lum and Chan (1997) showed that the problem has a W-shaped optimum when it is agreeably weighted, in the sense that $p_i > p_j$ implies $\alpha_i \le \alpha_j$ and $\beta_i \le \beta_j$ for all i, j. A schedule is said to be W-shaped with respect to the processing times if $p_j \ge p_k$ when $C_j < C_k < d$, and $p_j \le p_k$ when $d \le S_j < S_k$, where $S_j = C_j - p_j$ denotes the starting time of job j. In a W-shaped schedule, it is possible to have a job m with $S_m < d \le C_m$, called straddling job, such that $p_m > p_{m'}$ and $p_m > p_{m''}$, where jobs m' and m'' are, respectively, the jobs sequenced immediately before and after job m.

As a motivation for the above agreeable condition, Cai, Lum and Chan (1997) indicated that, in practice, a smaller job is charged at a higher rate, which in turn makes it relatively more important, and thus the penalty of missing its due date becomes heavier. They proposed two dynamic programming algorithms for the problem with the agreeable weight condition. The first one leads to an optimum in time $O(n^2 \sum p_j)$, and the second one can generate a sub-optimum in time $O(n \sum p_j)$. The second algorithm leads to an optimal solution in the special cases when a) $\alpha_i = \alpha_j$, $\beta_i = \beta_j$, $e_i = e_j$, $t_i = t_j$ for all i and j, or b) $\alpha_i = \alpha_j$, $\beta_i = \beta_j$ for all i and j, and $e_j = t_j$ for all j. An upper bound on the relative error of the sub-optimal solution was derived, and it was shown that, under certain conditions, the error tends to zero as n increases.

Computational results by Cai, Lum and Chan (1997) demonstrate the effectiveness of the proposed algorithms for the agreeably weighted problems up to 200 jobs in size, and show the possibility to use these algorithms as heuristic approaches for the general problem.

Federgruen and Mosheiov (1994) considered a special case of the above problem when $e_j = t_j = 0$, $\alpha_j = \beta_j = 1$, $j=1,\dots,n$, and proposed a greedy heuristic which requires $O(n^3)$ elementary operations and evaluations of the functions g and h. Their numerical experiments suggest that the greedy algorithm performs well under non-convex as well as convex functions g and h. For the same case of the problem, Kahlbacher (1993) proposed an $O(n\log n + n\sum_{j=1}^{n-1} p_j)$ pseudopolynomial algorithm. Another pseudopolynomial-time algorithm was proposed by Federgruen and Mosheiov (1993), who include a non-decreasing function $\gamma(d)$ into the objective function.

1.9. Seidmann, Panwalkar and Smith (1981) proposed another model of due date assignment where any due date d_j may be assigned to a job j, but there is a given common due date D, which represents the lead time that customers consider to be reasonable and expected. If a due date d_j is within this lead time, there is no penalty. However, each additional unit of due date beyond the reasonable lead time causes a penalty. So, the problem is to find the optimal values d_j^*, ($j = 1,...,n$), and an optimal sequence σ^* to minimize the total penalty

$$\sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma \max\{0, d_j - D\}),$$

where E_j and T_j are given by (1), and $\gamma > 0$.

For this model, Seidmann, Panwalkar and Smith (1981) found that SPT sequence is optimal and that the optimal due dates are given by $d_j^* = \sum_{i=1}^{j} p_i$, $j=1,...,n$, if $\gamma \le \beta$, and $d_j^* = \min\{D, \sum_{i=1}^{j} p_i\}$, $j=1,...,n$, otherwise.

De, Ghosh and Wells (1991a) formulated another problem involving the lead time D. The problem consists in finding a due date d, common to all the jobs, and a sequence σ to minimize the following objective function:

$$f(d,\sigma) = \sum_{j\in T} w_j + \gamma \max\{0, d - D\} = \sum_{j=1}^{n} w_j U_j + \gamma \max\{0, d - D\},$$

where T is the set of tardy jobs, $\gamma > 0$, $w_j > 0$, $j=1,...,n$, and $U_j = 0$ if $C_j \le d$ or $U_j = 1$ otherwise. For this problem, the order in which jobs are sequenced in each of the sets E (non-tardy jobs) and T is indifferent. Let M be the sum of the processing times of the jobs j with $w_j / p_j > \gamma$. Then, the problem is called restricted if $M > D$ and unrestricted if $M \le D$. De, Ghosh and Wells (1991a) showed that the restricted problem is NP-hard, proposed a solution based on a formulation of the problem as a knapsack problem, and gave a simple heuristic based on the continuous relaxation of the knapsack problem. A polynomial-time algorithm was presented for the unrestricted problem.

A more general objective function was considered by Kahlbacher and Cheng (1993): the problem is to find a common due date d^* and a sequence σ^* to minimize the objective function

$$f(d,\sigma) = \sum_{j\in T} w_j + \sum_{j\in E} h(d-C_j) + g(d),$$

where E and T are, respectively, the sets of early and tardy jobs, g and h are non-decreasing functions, $w_j > 0, j=1,...,n$. The problem is shown to be NP-hard even when a) $h(x)=0$ and $g(d)$ is an arbitrary function, or b) $g(d)=0$ and $h(x)$ is an arbitrary function. Kahlbacher and Cheng (1993) proposed polynomial-time algorithms in the following cases: a) $w_j = w$, $j=1,...,n$ (an $O(n\log n)$ algorithm), b) $h(x)=0$ and $g(d)=\gamma d$, $\gamma \geq 0$ (an $O(n)$ algorithm), c) $h(x)$ and $g(d)$ are linear (an $O(n^4)$ algorithm). It is interesting to note that for an externally given common due date (i.e., when the due date is not a decision variable), the problem with different w_j values is NP-hard even if $h(x)=0$ (due to Lawler and Moore, 1969, who proposed an equivalence between this problem and the knapsack problem which is NP-hard). As a consequence, the restricted variant of due date assignment problem (where the due date is restricted by a given upper bound) is NP-hard.

Lee, Danusaputro and Lin (1991) considered the problem which consists of finding a common due date d^* and the associated sequence σ^* to minimize the objective function

$$f(d,\sigma) = \sum_{j=1}^{n} (\alpha_j E_j + \beta_j T_j + w_j U_j),$$

where $U_j = 1$ if $C_j > d$ and 0 otherwise. The problem differs from the TWET problem considered in **1.5** by an additional fixed cost for each tardy job. The problem is considered under the assumption that $p_i/\alpha_i < p_j/\alpha_j$ implies $p_i/\beta_i < p_j/\beta_j$ for all i and j. A practical motivation for this condition is the expectation that, if job j is relatively more important than job i, both its earliness and tardiness weights will be greater than those of job j. Pseudopolynomial-time dynamic programming algorithms were proposed for both unrestricted (when $d \geq \sum p_j$) and restricted (when due date is upper bounded by D and $D < \sum p_j$) problems, and an $O(n^2)$ time algorithm was proposed in the special case of unrestricted problem with constant weights ($\alpha_j = \alpha$, $\beta_j = \beta$ and $w_j = w$ for all j) (Lee, Danusaputro and Lin, 1991).

1.10. Cheng (1991) considered the problem with constant due date assignment in a formulation which differs from the problems considered above: a due date d_j for job j is defined as $d_j = S_j + d$, where S_j is the time at which the machine starts processing job j, and d

is the constant flow allowance. So, in a sequence $\sigma = ([1],...,[n])$, the due date of the jth job will be $d_{[j]} = S_{[j]} + d = C_{[j-1]} + d$. For the problem which consists in minimizing

$$f(d,\sigma) = \sum_{j=1}^{n}(\alpha_{[j]}\left|C_{[j]} - d_{[j]}\right| + \gamma d),$$

where $0 \le \alpha_j \le 1, \sum_{j=1}^{n}\alpha_j = 1$, Cheng (1991) proposed a linear programming (LP) formulation and, by considering the LP dual problem, showed that the optimal flow allowance is independent of the job sequence and equal to one of the job processing times if $0 \le \gamma < 1/n$ or equal to $d^* = 0$ if $\gamma \ge 1/n$.

1.11. In industry, the shipping costs are significant cost factors. Therefore, the production model should include both the sequencing of the jobs and the scheduling of the deliveries. This is why it is useful to consider problems where three types of decisions are combined: scheduling, batching and due date assignment.

Hermann and Lee (1993) considered the WSAD problem (see **1.4**) with job batching and batch delivery costs. Under the similar introduction of job batching and batch delivery costs, Chen (1996) considered the problem of Panwalkar, Smith and Seidmann (1982) (see **1.2**). It is assumed that, after completion, the jobs are delivered to the customer in batches. There is no capacity limitation on the size of a batch (it may also include only one job). All early jobs (i.e., the jobs with $C_j < d$) are delivered in one batch at the common due date d without any delivery cost. Each batch with a tardy job (let us call it tardy batch) incurs a fixed delivery cost θ independent on the number of jobs in the batch. The delivery date of the batch is equal to the largest completion time among the jobs in this batch. Let D_j be the delivery date of job j. Then, earliness and tardiness of job j are defined, respectively, as $E_j = D_j - C_j$ and $T_j = D_j - d$. Let α and β be, respectively, the earliness and the tardiness penalty weights (identical for all the jobs). It is assumed that $\alpha \le \beta$. So, for an early job j, $E_j = d - C_j$ and its penalty is αE_j. A tardy job may incur both earliness and tardiness penalties if it is not delivered immediately upon its completion. The earliness and tardiness penalties of a tardy job j are $\alpha E_j = \alpha(D_j - C_j)$ and $\beta T_j = \beta(D_j - d)$. The objective function is:

$$f(\sigma,d,l) = \gamma d + \theta l + \sum_{j=1}^{n}(\alpha E_j + \beta T_j),$$

where l is the number of tardy batches. The goal is to find a due date d^*, a job sequence σ^* and a partition of the set of jobs into batches which jointly minimize this objective function.

It can be easily shown that each tardy batch is a set of consecutive tardy jobs in the job sequence σ and its delivery date is the completion time of the job completed last.

When the common due date is restrictive, i.e., $d < \sum_{j=1}^{n} p_j$, the problem is NP-hard even if $\theta = 0$ and $\alpha = \beta$: it follows from the NP-hardness of the restricted MAD problem (Hall, Kubiak and Sethi, 1991; see **1.3**). Hermann and Lee (1993) proposed a pseudopolynomial-time algorithm to find an optimal solution to the restricted variant of the problem with $\theta \neq 0$, $\gamma = 0$ and $\alpha \leq \beta$. Chen (1996) proposed an $O(n^5)$ dynamic programming algorithm for the unrestricted variant of the problem with $\theta \neq 0$, $\gamma \neq 0$ and $\alpha \leq \beta$.

Cheng and Kovalyov (1996) considered a problem with an objective function similar to the one considered by De, Ghosh and Wells (1991a) (see **1.9**) but under an assumption that jobs are grouped in n different sets. Each group j consists of $q_j \geq 1$ identical jobs with the same processing time $p_j \geq 0$ and the same weight $w_j \geq 0$. Each group will be partitioned into batches containing contiguously scheduled jobs. A set-up time s_j is required before a batch of group j is processed if it is processed first on the machine or immediately after a batch of another group. The machine can handle only one job at a time and cannot process any job when a set-up is performed. The objective is to determine a value for the common due date d and a schedule (i. e., the number of jobs in each batch and a processing order of the batches) so as to minimize

$$\sum_{j=1}^{n} w_j U_j + \gamma\, max\{0, d - D\},$$

where U_j is the number of tardy jobs of group j. The following property is useful to find an optimal solution.

<u>Property 7</u>. There exists an optimal schedule which consists of at most one early batch (i. e., $C_j \leq d$ for all jobs in this batch) and at most one tardy batch for each group.

Moreover, tardy batches can be scheduled in an arbitrary order after the last early batch, and, since there is a common due date, early batches can also be scheduled in an arbitrary order.

Cheng and Kovalyov (1996) showed that the problem is NP-hard in the following cases: a) $w_j = 1$, $p_j = 1$; b) $w_j = 1$, $s_j = s$; c) $q_j = 1$, $s_j = 0$; d) $q_j = 1$, $p_j = 0$, where $j=1,\dots,n$, and proposed two pseudopolynomial algorithms as well as a fully polynomial approximation scheme. In the special cases, when a) $w_j = 1$, $q_j = q$, b) $w_j = 1$, $p_j = p$,

$s_j = s$, or c) $q_j = q$, $p_j = p$, $s_j = s$, where $j=1,...,n$, $O(n\log n)$ algorithms were proposed. Kovalyov (1998) showed that the problem with $s_j = s$ and $p_j = p$ is NP-hard.

Azizoglu and Webster (1997) considered unrestricted TWET problem (see **1.5**) with the additional assumptions that each job belongs to one of f families, and that set-up time s_i is required in order to process a job of family i after a job in some other family. No set-up is required between jobs from the same family. A branch-and-bound algorithm was proposed which is effective at solving problems up to 15 jobs in size, and a beam search heuristic branch-and-bound procedure was applied to the problem. In this procedure, at each level of the branch and bound tree, only a limited number of nodes are selected for branching. By choosing this number of nodes (*beam width*), user can influence the trade-off between the time to obtain a solution and its quality.

With the same assumption of belonging jobs to f families, Chen, Li and Tang (1997) considered the problem of finding an optimal due date and the associated sequence of jobs to minimize the objective function

$$f(d,\sigma) = \alpha \sum_{j=1}^{n} U_j + \gamma d,$$

where α and γ are non-negative, and $U_j = 0$ if $C_j \le d$ and $U_j = 1$ otherwise. An $O(n^2)$ algorithm was proposed in the case when all jobs of each family are to be processed in one group one after another, and an $O(n\log n)$ algorithm was proposed in the case when jobs among families can be mixed together for processing.

1.12. Consider the problem of minimizing the maximal weighted absolute lateness (MWAL), i. e., of finding a due date d^* and the associated sequence σ^* to minimize the objective function

$$f(d,\sigma) = max_{1\le j\le n}\{w_j|C_j - d|\}.$$

Let d' be the best possible due date for a given sequence σ. Then, there exists an early job i and a tardy job k such that $w_k(C_k - d') = w_i(d' - C_i) = max_{1\le j\le n}\{w_j|C_j - d'|\}$ since, otherwise, it would exist a $\delta > 0$ such that either $d' + \delta$ or $d' - \delta$ is a better due date. Note that $w_k(C_k - d') = w_i(d' - C_i)$ implies $d' = (w_iC_i + w_kC_k)/(w_i + w_k)$. Thus, if we consider $d(i,k) = (w_iC_i + w_kC_k)/(w_i + w_k)$, the value of d' could be determined by checking all possible $d(i,k)$ in $O(n^2)$ time (Li and Cheng, 1994).

Unfortunately, finding an optimal sequence is difficult. Li and Cheng (1994) showed that the MWAL problem for a given d is NP-hard.

When $w_j = 1$ for all jobs, the problem of finding an optimal sequence becomes easy even for different given due dates. Lakshminarayan *et al.* (1978) proposed an $O(n\log n)$ algorithm to solve the problem. For the common due date assignment, this problem was considered by Cheng (1987b).

2. PARALLEL MACHINES

Consider the problem of scheduling n jobs on m parallel machines. Each job can be processed by any of m machines taking into account the following constraints: a machine performs at most one job at a time, and each job is performed at most by one machine at a time. If the machines are *identical*, they operate at the same speed, and processing time for job j is p_j. If the machines are *uniform*, each machine i has its own speed s_i, and processing time of job j on this machine will be $p_{ij} = p_j / s_i$. If the machines are *unrelated*, the speed of a machine is job-dependent, and processing time of job j on machine i is $p_{ij} = p_j / s_{ij}$.

The goal is to determine an optimal due date and to schedule the jobs on the machines, so as to minimize an objective function. The type of objective function will be specified below for each of the problems under consideration. In what follows, it is assumed that preemption is not permitted and all jobs become available for processing simultaneously at time zero.

2.1. In the case when the due date is given (i.e., is not a decision variable) and parallel machines are identical, the problem to minimize the mean absolute deviation (MAD) of completion times about a common due date d was considered by Sundararaghavan and Ahmed (1984), Hall (1986b) and Emmons (1987). In this problem, the objective function is $\sum_{j=1}^{n}|C_j - d| = \sum_{j=1}^{n}(E_j + T_j)$. For any schedule S, let n_i' and n_i'' be, respectively, the numbers of early jobs and tardy jobs on machine i, $i=1,\dots,m$. We denote by $[ji]$ the jth early job on machine i in schedule S, and by (ji) the jth (if counted from the last job) tardy job on machine i in S, $i=1,\dots,m$. Then, according to Emmons (1987), the objective function may be rewritten as

$$\sum_{i=1}^{m}\left(\sum_{j=1}^{n_i'}(j-1)p_{[ji]} + \sum_{j=1}^{n_i''} j p_{(ji)}\right).$$

Since at each machine the objective function is formulated as a sum of pairwise products of job processing times and of "positional weights", we have the similar situation as in the single machine problem considered in **1.2**. Therefore, the algorithm for parallel machines will be the following. Consider the jobs in the LPT order. Each job from the m jobs with the largest processing times should be placed first on each machine (the order of assigning this jobs to machines is arbitrary); the next $2m$ jobs with the largest processing times should be assigned in any order to the second and last positions; the next $2m$ jobs should occupy the third and the penultimate positions, etc. If the final group of jobs is less than $2m$, they can be assigned to any subset of the next $2m$ positions.

Emmons (1987) proposed a similar approach for the MAD problem with uniform machines and for the problem which consists of minimizing the objective function $\sum_{j=1}^{n}(\alpha E_j + \beta T_j)$ (the weighted sum of absolute deviation (WSAD) of completion times about a common due date) in the cases of identical or uniform machines.

For parallel machines, Kubiak, Lou and Sethi (1990) showed the equivalence of the WSAD problem and the problem of minimizing the mean flow time. On this basis, when parallel machines are unrelated, they proposed an algorithm for the WSAD problem by formulating the corresponding transportation problem.

For parallel identical machines, Webster (1997) studied the complexity of the MAD problem and the problem of minimizing the total weighted earliness and tardiness (TWET) with symmetric weights under the assumption that jobs belong to different families, and set-ups are required between the jobs from different families.

2.2. Emmons (1987) was the first to consider the parallel machine scheduling problems with due date assignment. For the MAD and WSAD problems with identical and uniform parallel machines, he proposed $O(n\log n)$ algorithms to find not only an optimal schedule but also an optimal due date. For identical machines, alternative optima are generated, and for uniform machines the algorithm gives a unique optimal solution where the value of the common due date is equal to $max_{1\leq i\leq m}\{\sum_{j=1}^{n_i'} p_{[ji]}/s_i\}$.

Emmons (1987) modified the algorithm for identical machines introduced in **2.1** so as to find an optimal due date considering the due date as a secondary criterion to be minimized. If $n \leq 4m$, this modification gives an optimal solution in polynomial time, otherwise it is a heuristic procedure.

The TWET problem, which is NP-hard in the ordinary sense in the single machine case (see **1.5**), is strongly NP-hard for parallel identical machines (in both restricted and unrestricted cases and even with symmetric weights) (Webster, 1997).

Alidaee and Panwalkar (1993) considered the MAD problem for the unrelated parallel machines and proposed an $O(n^3)$ algorithm for finding an optimal due date and the associated optimal schedule (by reducing the problem to a transportation problem).

Note that, for the above algorithms, the optimal due date is not always the minimal one. It is necessary to distinguish the problem of finding the optimal due date from the problem of finding the minimal one among optimal due dates. Unlike in the single machine case, these problems differ in complexity in the case of parallel machines.

De, Ghosh and Wells (1994b) showed that, for the identical parallel machines, the problem of finding the minimal due date and the associated optimal schedule in order to minimize the objective function $\sum_{j=1}^{n}(\alpha E_j + \beta T_j)$ is NP-hard even if $m = 2$, and strongly NP-hard for an arbitrary m. For the fixed m, they proposed a pseudopolynomial-time algorithm with a worst-case complexity $O(nm(2p_{max})^m)$, where p_{max} is the maximal processing time among the jobs. Alidaee and Panwalkar (1993) proposed a heuristic procedure to minimize makespan and to reduce the optimal common due date among optimal schedules with the objective function $\sum_{j=1}^{n}(E_j + T_j)$.

2.3. For identical parallel machines, Cheng and Chen (1994) and De, Ghosh and Wells (1994b) showed that the problem which consists of finding an optimal due date d* and the associated optimal schedule S* with respect to the objective function

$$f(S,d) = \sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d)$$

is NP-hard even if $m = 2$. Remind that, for a single machine, this problem is polynomially solvable (see **1.2**).

The following properties are valid for this problem (as well as for the problems considered in **2.2**).

Property 8. In an optimal schedule, there is no idle time between jobs on any machine, and the sequence of jobs on each machine is V-shaped about the due date: jobs completed by the due date are in LPT order, and jobs completed after the due date are in SPT order.

Property 9. In an optimal schedule, there exists at least one job j such that $C_j = d*$.

For any schedule S, let $z*(S) = min_{d\geq 0}\{f(S,d)\}$ and $d*(S) = min_{d\geq 0}\{d | f(S,d) = z*(S)\}$. Given a schedule S, the value $d*(S)$ can be easily computed. Let $C_{[K]}$ be the Kth smallest completion time in S, n_0 be the number of jobs in S with completion time equal to $C_{[K]}$, and $l = n(\beta - \gamma)/(\alpha + \beta)$. Then, $d*(S) = C_{[K]}$, where K satisfies the inequality $l \leq K \leq l + n_0$. This result is similar to Property 2 in the single machine case.

Cheng (1989) proposed a heuristic algorithm for this problem, De, Ghosh and Wells (1991b) characterized necessary conditions for an optimal schedule. For an arbitrary m, the problem was shown to be strongly NP-hard, while for the fixed m a pseudopolynomial-time algorithm was proposed (De, Ghosh and Wells, 1994b).

In the following special cases of this problem, polynomial-time algorithms are proposed: a) $\gamma = 0$ (Emmons, 1987) (see **2.2**), b) $p_j = p$, $j=1,\dots,n$ (Cheng and Chen, 1994), c) $\gamma \geq \beta$; in this case the problem is equivalent to a $P|\,|\sum C_j$ problem (according to the notations of Lawler *et al.*, 1993). In cases a) and c), the complexity of the algorithms is $O(n \log n)$, while it is $O(1)$ in case b).

Adamopoulos and Pappis (1998) proposed a heuristic procedure for the problem with the same objective function in case of unrelated parallel machines.

2.4. Kahlbacher and Cheng (1993) considered the problem of assigning due date and scheduling jobs on identical machines in order to minimize the objective function

$$f(d,S) = \sum_{j \in E} h(d - C_j) + \sum_{j \in T} w_j + g(d),$$

where E and T are, respectively, the sets of early and tardy jobs for the schedule S; $w_j > 0$ is a weight associated with job j, $j=1,\dots,n$; g and h are real valued, non-decreasing functions defined on the set of real, non-negative numbers so that $g(0)$ and $h(0)$ are equal to 0.

This problem for an externally given due date is NP-hard (Kahlbacher and Cheng, 1993) even if $h(x) = 0$ and $w_j = 1$, $j=1,\dots,n$ (it may be also regarded as the restricted due date assignment problem). When the due date can be arbitrarily assigned, polynomial-time algorithms were proposed for linear function $h(x)$ and $g(d) = 0$ (complexity is $O(n \log n)$ if $w_j = w$, $j=1,\dots,n$, and $O(n^4)$ if values of w_j are different) (Kahlbacher and Cheng, 1993). The problem was shown to be NP-hard (even if $w_j = w$, $j=1,\dots,n$) for a) linear $g(d)$ and $h(x) = 0$, and b) arbitrary $h(x)$ and $g(d) = 0$.

2.5. Cheng and Chen (1994) showed that the problem of finding a due date and the associated schedule which minimize the objective function

$$f(d,S) = max\{max_{1 \leq j \leq n} E_j, max_{1 \leq j \leq n} T_j\}$$

is NP-hard for parallel identical machines. Remind that, in a single machine case, this problem is polynomially solvable (see **1.12**).

When each job has a weight w_j, the problem (which we refer in **1.12** as the MWAL problem; it consists in minimizing maximal weighted lateness) becomes strongly NP-hard (Li and Cheng, 1994).

3. CONCLUSION

In this paper, we have presented a review on scheduling jobs on a single machine and parallel machines under a common due date which is a decision variable. The SLK, TWK, NOP and PPW due date assignment models and the assignment of positional due dates will be considered in another paper which may be regarded as the second part of this paper.

The results for the common due date assignment and scheduling problems are summarized in Table 1. We adopt the standard three-field notation $a|b|c$ used for scheduling problems (Lawler *et al.*, 1993), where a describes machine environment and specifies the number of machines, b describes schedule and job characteristics, and c describes the optimality criterion. We extend this notation it in the following way. The notation $d_j = d$ denotes the (unrestricted) problem with a common due date assignment, and $d_j = d^{res}$ denotes the restricted version of this problem. The notation $d_j = d(\pm e)$ denotes the unrestricted version of the problem with no penalty cost within interval $[d-e, d+e]$ around a common due date d. The notation $d_j = d^{min}$ denotes the problem to find minimal due date among the optimal ones. For the parallel machines, as usual, notations P, Q and R denote, respectively, identical, uniform and unrelated machines, while Pm means that the number of identical machines is fixed and equal to m. In the column **Complexity**, notation P denotes polynomially solvable problem. In the column **Algorithm**, notation PsP denotes pseudopolynomial-time algorithm, notations Enumer., Heurist. and Approx. denote enumerative, heuristic and approximation procedures, respectively. We do not include in the table the results related to the special cases of the problems and to the special additional conditions (such as agreeable weights, processing in batches, etc.), which are described above. The results for the general model introduced in **1.8** are not included in the table since the notations for the objective function require additional explanations.

Table 1. CON due date assignment and scheduling

	Problem	Complexity	Algorithm
1	$1\|d_j = d\|\sum(E_j + T_j)$	P	$O(n \log n)$ Kanet (1981), Panwalkar *et al.* (1982), Bagchi *et al.* (1986), Hall (1986b), Bagchi, Chang&Sullivan (1987)
2	$1\|d_j = d^{res}\|\sum(E_j + T_j)$	NP-hard Hall, Kubiak&Sethi (1991) Hoogeveen&Van de Velde (1991)	PsP Hall, Kubiak&Sethi (1991), Ventura& Weng (1995), De *et al.* (1993) Enumer. Bagchi *et al.* (1986), Szwarc (1989) Heurist. Sundararaghavan&Ahmed (1984) Approx. Hoogeveen *et al.* (1994)
3	$1\|d_j = d\|\sum(\alpha E_j + \beta T_j)$	P	$O(n \log n)$ Panwalkar *et al.* (1982), Bagchi, Chang&Sullivan (1987)
4	$1\|d_j = d^{res}\|\sum(\alpha E_j + \beta T_j)$	NP-hard (from 2)	PsP De *et al.* (1993) Enumer. Bagchi, Chang&Sullivan (1987)
5	$1\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma d)$	P	$O(n \log n)$ Panwalkar *et al.* (1982)
6	$1\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma\, max\{0, d_j - D\})$	P	$O(n \log n)$ Seidmann *et al.* (1981)
7	$1\|d_j = d\|\sum \alpha_j(E_j + T_j)$	NP-hard Hall&Posner (1991)	PsP Hall&Posner (1991) Enumer. (Cheng, 1990) Approx. (Hall&Posner, 1991) Heurist. (Hao *et al.*, 1996)
8	$1\|d_j = d^{res}\|\sum \alpha_j(E_j + T_j)$	NP-hard (from 2)	PsP Hoogeveen &Van de Velde (1991)
9	$1\|d_j = d\|\sum(\alpha_j E_j + \beta_j T_j)$	NP-hard (from 7)	Enumer. Gupta *et al.* (1990), Dileepan (1993), De *et al.* (1994a) Heurist. Dileepan (1993), De *et al.* (1994a), James (1997)
10	$1\|d_j = d^{res}\|\sum(\alpha_j E_j + \beta_j T_j)$	NP-hard (from 2)	Heurist. James (1997)

Table 1. CON due date assignment and scheduling (continuation)

	Problem	Complexity	Algorithm
11	$1\|d_j = d(\pm e)\|\sum(E_j + T_j)$	P	$O(n \log n)$ Cheng (1988), Weng&Ventura (1994), Wilamowsky *et al.* (1996)
12	$1\|d_j = d\|\sum(E_j^2 + T_j^2)$	NP-hard Kubiak (1993)	PsP Kahlbacher (1989), De *et al.* (1992,1993) Enumer. Bagchi, Sullivan&Chang (1987) Heuristic. Gupta *et al.* (1993)
13	$1\|d_j = d^{res}\|\sum(\alpha E_j^2 + \beta T_j^2)$	NP-hard (from 12)	PsP Kahlbacher (1989,1993), De *et al.* (1993) Enumer. Bagchi, Chang&Sullivan (1987), Bagchi, Sullivan&Chang (1987), De *et al.* (1989,1990)
14	$1\|d_j = d\|\sum(w_j U_j + \gamma\, max\{0, d - D\})$	P	$O(n)$ De *et al.* (1991a)
15	$1\|d_j = d^{res}\|\sum(w_j U_j + \gamma\, max\{0, d - D\})$	NP-hard from Lawler&Moore (1969) De *et al.* (1991a)	Enumer. De *et al.* (1991a) Heuristic. De *et al.* (1991a)
16	$1\|d_j = d\|\sum w_j U_j + \gamma d$	P	$O(n)$ Kahlbacher&Cheng (1993)
17	$1\|d_j = d\|\sum U_j + \sum h(E_j) + g(d)$	P	$O(n \log n)$ Kahlbacher&Cheng (1993)
18	$1\|d_j = d\|\sum w_j U_j + \sum \alpha E_j + \gamma d$	P	$O(n^4)$ Kahlbacher&Cheng (1993)
19	$1\|d_j = d\|\sum w_j U_j + g(d)$	NP-hard Kahlbacher& Cheng (1993)	
20	$1\|d_j = d\|\sum w_j U_j + h(E_j)$	NP-hard Kahlbacher& Cheng (1993)	
21	$1\|d_j = d\|max(E_{max}, T_{max})$	P	$O(n^2)$ Cheng (1987b)
22	$1\|d_j = d\|max_j(w_j\|L_j\|)$	NP-hard Li&Cheng (1994)	

Table 1. CON due date assignment and scheduling (continuation)

	Problem	Complexity	Algorithm
23	$Q\|d_j = d\|\sum(E_j + T_j)$	P	$O(n\log n)$ (Emmons, 1987)
24	$Q\|d_j = d\|\sum(\alpha E_j + \beta T_j)$	P	$O(n\log n)$ (Emmons, 1987)
25	$R\|d_j = d\|\sum(E_j + T_j)$	P	$O(n^3)$ Alidaee&Panwalkar (1993), Kubiak *et al.* (1990)
26	$P2\|d_j = d^{min}\|\sum(E_j + T_j)$	NP-hard De *et al.* (1994b)	PsP De *et al.* (1994b)
27	$Pm\|d_j = d^{min}\|\sum(\alpha E_j + \beta T_j)$	NP-hard (from 26)	PsP De *et al.* (1994b)
28	$P\|d_j = d^{min}\|\sum(E_j + T_j)$	strongly NP-hard De *et al.* (1994b)	Heuristic. Alidaee&Panwalkar (1993)
29	$P\|d_j = d, p_j = p\|\sum(\alpha E_j + \beta T_j + \gamma d)$	P	$O(1)$ (Cheng&Chen, 1994)
30	$P2\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma d)$	NP-hard Cheng&Chen (1994), De *et al.* (1994b)	PsP De *et al.* (1994b)
31	$Pm\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma d)$	NP-hard (from 30)	PsP De *et al.* (1994b)
32	$P\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma d)$	strongly NP-hard De *et al.* (1994b)	Heuristic. Cheng (1989)
33	$Q\|d_j = d\|\sum(\alpha E_j + \beta T_j + \gamma d)$	strongly NP-hard (from 32)	Heuristic. Adamopoulos&Pappis (1998)
34	$P\|d_j = d\|\sum U_j + \sum \alpha E_j$	P	$O(n\log n)$ Kahlbacher&Cheng (1993)
35	$P\|d_j = d\|\sum w_j U_j + \sum \alpha E_j$	P	$O(n^4)$ Kahlbacher&Cheng (1993)
36	$P\|d_j = d\|\sum U_j + \gamma d$	NP-hard Kahlbacher&Cheng (1993)	
37	$P\|d_j = d\|\max(E_{max}, T_{max})$	NP-hard Cheng&Chen (1994)	
38	$P\|d_j = d\|\max_j(w_j\|L_j\|)$	strongly NP-hard Li&Cheng (1994)	

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